

Maximizing the Profit of a Business

Math 1010 Intermediate Algebra Group Project

2/25

In this project your group will solve the following situation:

A manufacturer produces the following two items: computer desks and bookcases. Each item requires processing in each of two departments. Department A has 55 hours available and department B has 39 hours available each week for production. To manufacture a computer desk requires 4 hours in department A and 3 hours in department B while a bookcase requires 3 hours in department A and 2 hours in department B. Profits on the items are \$72 and \$23 respectively. If all the units can be sold, how many of each should be made to maximize profits?

Let X be the number of computer desks that are sold and Y be number of bookcases sold.

- Write down a linear inequality for the hours used in Department A

$$55 \geq 4x + 3y$$

$$\frac{55}{4} = \frac{4x}{4} \quad \boxed{X = 13.75}$$

$$\frac{55}{3} = \frac{3y}{3} \quad \boxed{Y = 18.3}$$

- Write down a linear inequality for the hours used in Department B

$$39 \geq 3x + 2y$$

$$\frac{39}{3} = \frac{3x}{3} \quad \boxed{X = 13}$$

$$\frac{39}{2} = \frac{2y}{2} \quad \boxed{Y = 19.5}$$

There are two other linear inequalities that must be met. These relate to the fact that the manufacturer cannot produce negative numbers of items. These inequalities are as follows:

$$\begin{aligned} X &\geq 0 & 55 &= 3y & Y &= 18.3 \\ Y &\geq 0 & 39 &= 3x & X &= 13 \end{aligned}$$

- Next, write down the profit function for the sale of X desks and Y bookcases:

$$P = 72x + 23y$$

You now have four linear inequalities and a profit function. These together describe the manufacturing situation. These together make up what is known mathematically as a **linear programming** problem. Write all of the inequalities and the profit function together below. This is typically written one on top of another, with the profit function last.

$$55 \geq 4x + 3y$$

$$39 \geq 3x + 2y$$

$$x \geq 0$$

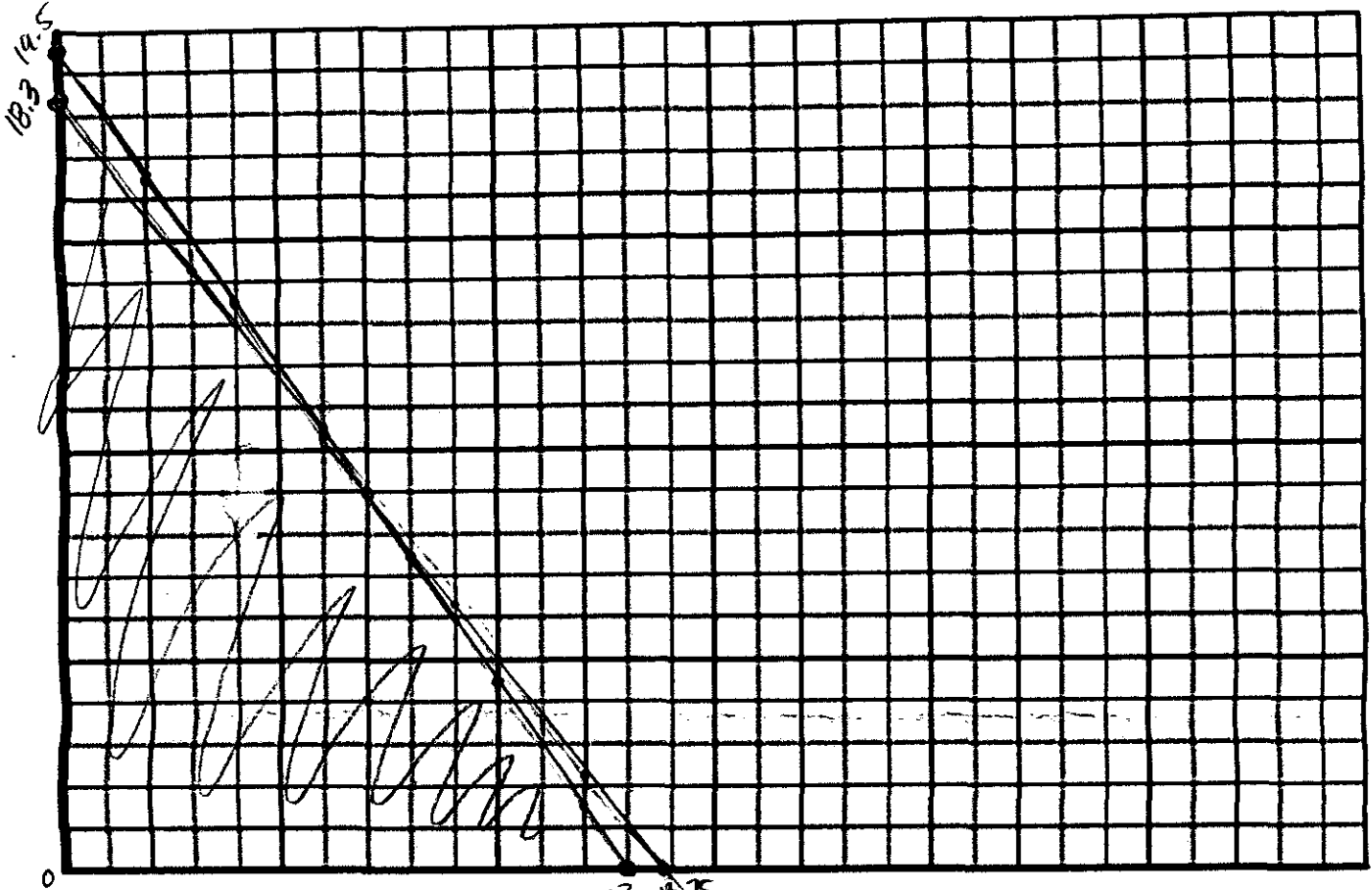
$$y \geq 0$$

$$P = 72x + 23y$$

$4x + 3y \leq 55$ BETTER

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4. To solve this problem, you will need to graph the **intersection** of all four inequalities on one common XY plane. Do this on the grid below. Have the bottom left be the origin, with the horizontal axis representing X and the vertical axis representing Y.



$$55 \geq 4(0) + 3(0)$$

$$55 \geq 0$$

True

$$39 \geq 3(0) + 2(0)$$

$$39 \geq 0$$

True

5. The above shape should have 4 corners. Find the coordinates of the ordered pairs that make up these corners. For the intersection of the two slanted lines you will have to solve the 2 by 2 system made up of their equations. $(0, 0)$ $(0, 18.3)$ $(13, 0)$ $(7, 9)$

$$\begin{array}{r}
 -3 \quad -2 \\
 4 \quad 3
 \end{array}
 \left\{ \begin{array}{l}
 55 \geq 4x + 3y \\
 39 \geq 3x + 2y
 \end{array} \right.
 =
 \begin{array}{l}
 -110 \geq -8x + 6y \\
 117 \geq 9x + 6y
 \end{array}
 \quad
 \begin{array}{l}
 -165 \geq -12x + 9y \\
 156 \geq 12x + 8y
 \end{array}$$

$$\begin{array}{l}
 \frac{7}{1} \geq \frac{1x}{1} \\
 \boxed{7 \geq x}
 \end{array}
 \quad
 \begin{array}{l}
 \frac{-9}{-1} \geq \frac{-1y}{-1} \\
 \boxed{9 \geq y}
 \end{array}$$

6. The next thing to do is to plug each of the points you found in part 5 into the profit function to determine which ordered pair gives the maximum profit. Do this and write a sentence describing how many of each type of furniture you should build and sell and what is the maximum profit you will make.

$(0, 0)$ B

$$P = 72x + 23y$$

$$\begin{array}{lll}
 P = 72(13) + 23(0) & P = 72(0) + 23(18.3) & P = 72(7) + 23(9) \\
 P = 926 & P = 421.66 & P = 504 + 207 \\
 & & P = 711
 \end{array}$$

$(7, 9)$

To make the maximum profit, you would have to sell at least 9 computer desks and at least 8 book cases. The maximum profit would be \$711.00

-2

B

7. Reflective Writing.

Did this project change the way you think about how math can be applied to the real world? Write one paragraph stating what ideas changed and why. If this project did not change the way you think, write how this project gave further evidence to support your existing opinion about applying math. Be specific.